6 Hindered systems and rheology

On increasing the particle concentration from the low values considered in the previous chapter, the system properties will change considerably from those of the continuous phase, usually water, and the individual particles. In settling, the presence of a large number of fine particles will *hinder* the fall of the larger particles and the very small particles will be dragged down more quickly than under free settling. There is an important question to be considered in our treatment of high concentration suspensions: are we interested in the behaviour of the particles compared to the continuous phase, such as during hindered settling, or the behaviour of the suspension as a new homogeneous phase, such as during pumping of a mixture? The latter is the concern of buoyancy, viscosity and rheology and is considered in the later sections. Initially, the settling of particles within a continuous phase will be considered. Another fundamental concern is the particle concentration at which hindered systems are appropriate, rather than the models discussed in the previous chapter. In general, hindered settling is appropriate when the particle concentration is greater than about 1% by mass. To describe concentrated suspensions we will need to use some of the definitions from Chapter 3. An illustration of porosity and solid concentration by volume fraction is reprinted in Figure 6.1.

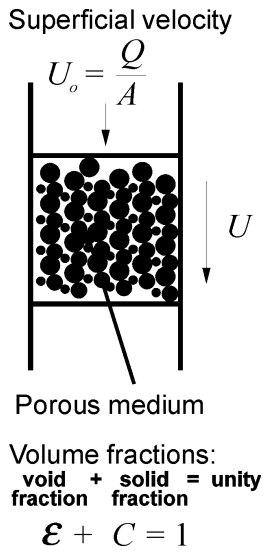


Fig. 6.1 Illustration of porous medium – including settling suspension

The industrial equipment in which hindered settling is conducted is simply tanks, which may be operated batch-wise or continuously. Settling is a cheap method of concentrating solids, the driving potential for it is free (gravity), but it provides only a limited final solid concentration and the process is slow. However, it is very often used in *thickening* a suspension before a more capital intensive operation, such as filtration. The design of industrial thickeners is covered in Section 6.3.

6.1 Hindered settling and zone theory

One method to ascertain if a suspension is settling in the hindered settling regime is to mix the suspension thoroughly and to watch it settle in a laboratory measuring cylinder. If an interface between the settling suspension and clearer residual liquid is apparent, then the settling is within the hindered settling regime. It may be possible to observe the rate of descent of the interface with time, as illustrated in Figure 6.2. Below the settling interface exists a porous medium, similar to that illustrated in Figure 6.1. If it were possible to turn the measuring cylinder upside-down, without everything falling out, then the liquid velocity upwards, required to keep the settling interface stationary, would be the same as the superficial velocity illustrated in Figure 6.1. Hence, the settling velocity is the same as the superficial velocity and we can justifiably use the symbol U_o for both.

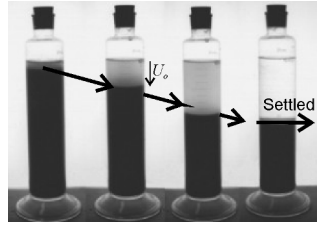


Fig. 6.2 Hindered settling in cylinders – interface fall with time

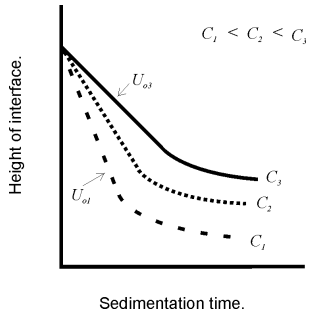


Fig. 6.3 Variation of settling velocity with starting concentration

The technical term for the residual liquid above the interface is the *supernatant* and, for the duration of this chapter, it will be assumed that it is entirely free of solids. It will also be assumed that there are no wall support effects for the settling suspension, a vessel of diameter equal to, or greater than, 150 mm is often recommended for this purpose, but tests in different diameter vessels can show if this effect is important.

The hindered settling velocity is a strong function of the particle concentration and, for a given material, nothing else; i.e. it is independent of vessel diameter, shape, etc. The dependency on concentration is logical: at the highest possible concentration no settling can take place, at low concentration the particle will settle under free settling conditions. Thus, in hindered settling the settling velocity will be between these two limits and a mathematical expression for hindered settling velocity could be considered to be a correction term to the free settling velocity (U_t). The most famous empirical relation between settling velocity (U_o) and solid concentration by volume fraction (C) is the *Richardson and Zaki* equation

$$U_o = U_t (1 - C)^n \tag{6.1}$$

where n is a variable constant that depends on the Particle Reynolds number and may be dependent on vessel diameter (d):

Particle Reynolds number	n for small tubes	n for large tubes
< 0.2	4.65 + 19.5 x/d	4.65
0.2 < Re' <1	(4.35 + 17.5 x/d) Re'$^{-0.03}$	4.35 Re'$^{-0.03}$
1 < Re' < 200	(4.45 + 18 x/d) Re'$^{-0.1}$	4.45 Re'$^{-0.1}$
200 < Re' < 500	4.45 Re'$^{-0.1}$	4.45 Re'$^{-0.1}$
Re' > 500	2.39	2.39

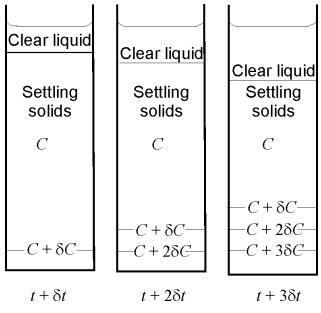


Fig. 6.4 Solid concentration increases starting at the base and apparently rising upwards in the vessel

In Figure 6.3 all the settling interface plots are straight lines, followed by curves. This can be explained by considering what takes place within the settling suspension. At the start, the concentration is C_f and uniformly distributed within the vessel. At a time δt the concentration at the base of the vessel is $C_f + \delta C$. At the next instance in time, $2\delta t$, the concentration at the base is $C_f + 2\delta C$. If we were to track the concentration $C_f + \delta C$ we would find that it is now slightly higher up the vessel than the base. Thus layers of constant concentration appear to *propagate* upwards. Of course, all the solids are settling; no solids are propagating upwards – we are merely looking for a region of a given concentration of solids. It takes some time before the solid concentration below the settling interface increases from that of the original and, according to equation (6.1), settling velocity is a unique function of concentration; hence, the settling velocity must remain constant, giving rise to the initial straight lines in Figure 6.3. The concentration increases from the base are illustrated in Figure 6.4.

In our settling suspension we can see that there are zones, or regions, of: supernatant liquid, the original concentration (just below the settling interface) and it is logical that there will be another zone consisting of fully settled sediment at the base of the vessel. There is a fourth zone: that of variable (increasing) concentration lying between the sediment and initial concentration zones. This is illustrated on Figure 6.5 and the mathematical description of the concentrations in this zone was first published by Kynch in the 1951, and is covered in Section 6.4. At this stage, it is reasonable to suggest that because the settling velocity is a unique function of concentration the velocity at which the concentrations propagate upwards will be also, giving rise to lines emanating from the origin called concentration *characteristics*.

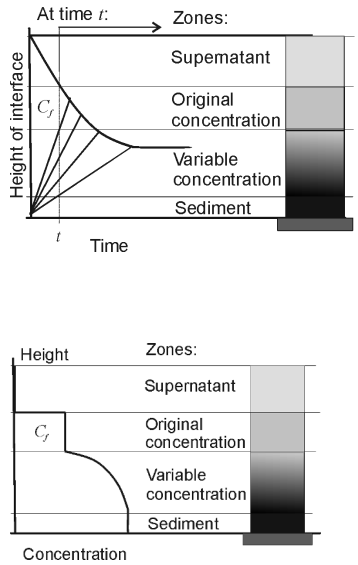


Fig. 6.5 Zones within a settling suspension at a selected time t

6.2 Batch settling flux

When a characteristic reaches the settling interface the concentration at the interface will become the value of the characteristic and, in accordance with equation (6.1), the settling velocity will become the value given by that concentration. This leads to an increasing concentration at the interface, after the end of the constant settling rate period, hence the settling rate decreases with time.

The height of interface against time plot, as illustrated in Figure 6.3, is very simple to obtain from experimental data and is very powerful in the information that can be deduced from it. Design information is concerned with the ability to pass a required mass of solids per unit area and time; i.e. kg m^{-2} s^{-1}, or mass of solids per unit time kg s^{-1}, this is the *solids flux*. Mathematically, the solids flux (kg s^{-1}) due to settling in a batch vessel is

$$G = U_o C A \rho_s \tag{6.2}$$

However, in most cases the vessel area and the solids density are constant; hence, the solids flux is usually abbreviated to (G')

$$G' = U_o C \tag{6.3}$$

which has the units m s^{-1}. A batch flux curve, Figure 6.6, can be plotted by conducting several sedimentations at different concentrations and measuring the initial settling velocities at the concentrations used, as illustrated in Figure 6.3. The batch flux curve possesses a maximum and this can be explained by consideration of the two limits: at zero concentration the flux must be zero, in accordance with equation (6.3), and at the highest possible concentration the flux will again be zero because the settling velocity term will be zero. Between these two extremes the flux will have finite values; hence, the batch flux curve must possess a maximum, as illustrated in Figure 6.6.

The knowledge that concentration characteristics reach the settling interface, and then the interface settles at the velocity for that concentration, can be used to determine the settling velocities for

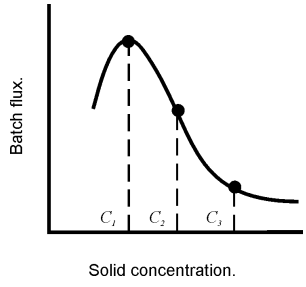


Fig. 6.6 The batch flux curve

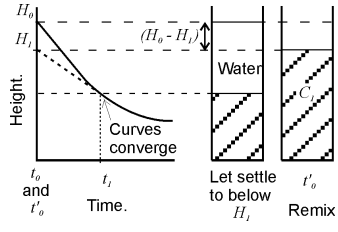


Fig. 6.7 Construction to transform a settling curve

concentrations greater than that used in the original suspension. The justification comes from a material balance as no solids are lost

$$C_{\mathrm{f}} H_{\mathrm{o}} A \rho_{\mathrm{s}} = C_1 H_1 A \rho_{\mathrm{s}} \tag{6.4}$$

where H_{o} represents the full height of the suspension before settling commences. Thus it is possible to mix the suspension and obtain a settling velocity at concentration C_{f}; then after some time to remove some supernatant liquid and fully mix the suspension again and let it settle. This will give a slower settling velocity and the solid concentration of the fully mixed suspension can be deduced from equation (6.4) by rearrangement to obtain the new initial concentration

$$C_1 = \frac{H_o}{H_1} C_{\mathrm{f}} \tag{6.5}$$

Clearly, it is physically possible to perform the experiment to obtain the settling rate at the new concentration, but it is also possible to perform the transformation of the data graphically, without the need of the experiment, as the two settling curves are identical after the point at which the concentration characteristic reaches the interface. This is illustrated in Figure 6.7.

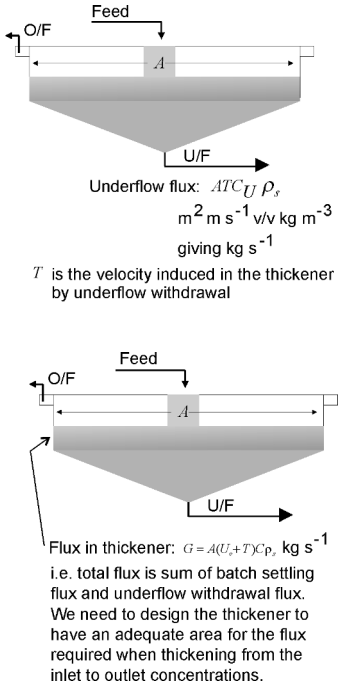


Fig. 6.8 Fluxes in a thickener

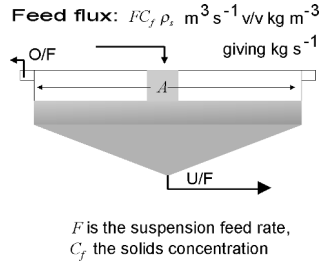


Fig. 6.9 Flux (kg s^{-1} of solids) fed to the thickener

6.3 Thickener design

A continuous thickener is a vessel with a feed, at low solid concentration, and two output streams: an overflow of clean liquid and an underflow suspension of much greater concentration than the feed. The vessel is normally circular, with a conical bottom that is raked to bring the solids into the discharge well. The design requirement is to deduce the plan area required for a given flow rate of solids entering and to achieve the desired degree of thickening. If insufficient area is provided then the concentration of solids within the vessel increases and will eventually leave in the overflow. The fluxes within a thickener include the batch flux, described above, but there is an additional flux due to the continual removal of material from the base; i.e. underflow. The fluxes are illustrated in Figure 6.8, which includes a schematic diagram of the thickener.

The thickener feed flux is illustrated in Figure 6.9 and this is the mass feed rate entering the system. Equating all the flux terms provides, where F is the thickener feed rate (m^3 s^{-1})

$$G = A(U_{\mathrm{o}} + T)C\rho_{\mathrm{s}} = FC_{\mathrm{f}}\rho_{\mathrm{s}}$$

i.e. the feed flux must be equal to the batch and underflow withdrawal flux within the thickener, thus

$$A = \frac{FC_{\mathrm{f}}}{(U_{\mathrm{o}} + T)C} \tag{6.6}$$

The flux at an arbitrary height within the thickener will be equal to the flux at the underflow (TC_{u}), which can be substituted into equation (6.6)

$$A = \frac{FC_f}{(U_o + T)C_u} \tag{6.7}$$

but the batch flux at the underflow will be negligible compared to the underflow withdrawal flux; hence we can write

$$A = \frac{FC_f}{TC_u} \tag{6.8}$$

Equation (6.8) is the design equation and to use it we must determine the flux at the underflow concentration.

The underflow withdrawal induces a downward velocity within the thickener which is constant for all the concentrations present from C_f to C_u. Hence, the underflow withdrawal flux is a straight line on a graph of flux against concentration, see Figure 6.10. Adding the batch and underflow withdrawal flux together gives the *composite flux curve*; which has a minimum at a *critical concentration* between C_f and C_u. It is this concentration that has the minimum solids flux, or handling, ability. Hence, if too much solids are added to the thickener the critical concentration (C_c) will build up within the device and eventually overflow. The batch, underflow and composite flux curves are illustrated in Figure 6.11.

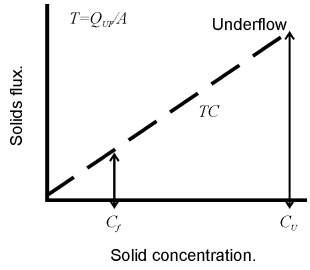


Fig. 6.10 Underflow withdrawal flux

In Figure 6.12 the construction required to determine TC_u, for use in equation (6.8), is illustrated. The minimum composite flux occurs at C_c and is numerically equal to the underflow flux at C_u; i.e. TC_u. However, the composite flux at the critical concentration does have two components: that due to batch settling and underflow withdrawal, which are marked on the figure. Clearly, it is possible for the thickener to be operated under conditions that require a batch flux less than the value at the batch flux curve, at C_c, but batch fluxes greater are not possible. Hence, the limit of operation is where C_c meets the batch flux curve. So, for a required underflow concentration (C_u) a line drawn through the concentration axis at C_u and tangential to the batch flux curve will meet the flux curve at the value of TC_u. This value can then be used in equation (6.8) to determine the thickener area.

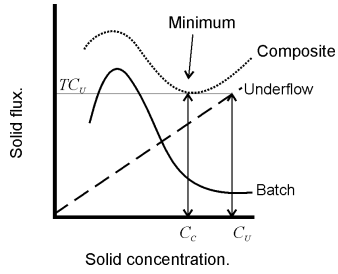


Fig. 6.11 All the flux curves

Thickener height is not determined by flux theory and, in general, thickeners are short vessels with diameters up to 50 metres. Minimum heights are allocated for the raked zone (0.5 m), solids storage zone (0.5 m) and clarification (0.5 m). Thus, a thickener is usually 1.5 to 4 m in height, unless solids compression is important.

An alternative design method to the use of the batch flux curve construction described above was originally described by *Coe and Clevenger*. A flux balance between the feed and underflow provides

$$G = ATC_u\rho_s = FC_f\rho_s \quad \text{hence} \quad T = \frac{FC_f}{AC_u}$$

which can be substituted in to equation (6.7) and rearranged to give

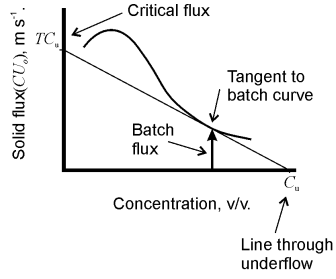


Fig. 6.12 Construction to determine the critical flux – equation (6.8)

$$A = \frac{FC_f}{U_o}\left[\frac{1}{C} - \frac{1}{C_u}\right] \tag{6.9}$$

Recommendation
Leave Sections 6.4 and 6.5 until after completing the problems.

Equation (6.9) is solved by selecting concentrations between C_f and C_u, where U_o is required for the value of C selected, and using the greatest area for the design. Equation (6.9) is easier to apply than the graphical technique described earlier, but equation (6.8) and the graphical construction has the advantage that it can be used to predict the underflow concentration from an existing thickener under different operating loads; i.e. FC_f.

6.4 Kynch analysis

Figure 6.13 includes a slice through a settling suspension and illustrates a batch settling curve. A mass balance on the solid slice gives, in terms of kg s^{-1} of solids:

input $\quad CU_o A\rho_s$

output $\quad \left[CU_o + \frac{d(CU_o)}{dz}\delta z\right]A\rho_s$

accumulation $\quad \frac{dC}{dt}A\rho_s\delta z$

where z is the vertical co-ordinate in the batch settling vessel. Using input - output gives accumulation, taking to an infinitesimal distance, cancelling and rearranging gives

$$\frac{dz}{dt} = -\frac{d(CU_o)}{dC} \tag{6.10}$$

i.e. the rate of increase in height (dz/dt) of a concentration C is the differential of the product of the concentration and the settling velocity, which could be calculated from equation (6.1). Equation (6.10) shows that the propagation velocity will be a unique function of solid concentration, a consequence of the settling velocity, equation (6.1), being a unique function. Thus, equation (6.10) shows mathematically that the propagation velocity for a given concentration will be a constant; hence, the straight lines from the origin to the point where they meet the settling interface curve. Equations (6.10) and (6.3) show that the propagation rate of a concentration characteristic can be obtained from the tangent, or differential, to the batch flux curve drawn at that concentration value.

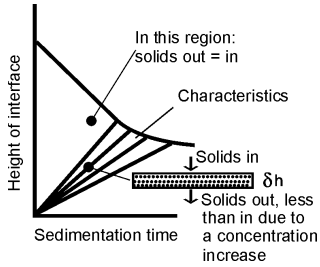


Fig. 6.13 A lamina layer within a settling suspension and the batch settling curve

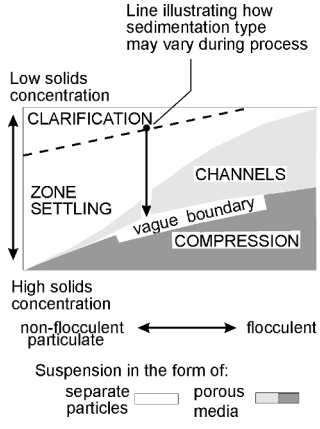


Fig. 6.14 Diagram showing types of sedimentation

6.5 Compressible sediments

The various types of settling are illustrated in Figure 6.14. It is called Fitch's *paragenesis* diagram. Clarification was described in Chapter 5 and the earlier part of this chapter discussed zone settling, which occurs when each particle behaves in an incompressible manner and is free to move within the suspension, subject to undergoing hindered settling. Flocculent sedimentation is when the particles are stuck together, either by a synthetic flocculent or by natural

aggregation. This leads to compressible sediments. Compression during sedimentation is sometimes called *self weight filtration*, and the compressible equations (4.15) and (4.16) can be applied. During compression a solids stress gradient exists (dP_s/dz), this gradient is not present in zone settling. A force balance, neglecting inertia, on a lamina settling layer gives (stress = solids weight – liquid drag)

$$\frac{dP_s}{dz} = Cg(\rho_s - \rho) - \frac{\mu}{k}U_o \tag{6.11}$$

where k is the permeability. For a more detailed discussion on this topic the interested reader is directed to more advanced texts – see the further reading section.

6.6 Homogeneous systems

The earlier sections considered particles sedimenting in a continuous phase, such as water. The remaining sections cover the treatment of homogeneous systems, where the particles do not separate from the continuous phase, but their existence leads to modification of the properties of the continuous phase such as buoyancy and viscosity. These considerations are, therefore, relevant to situations when the suspension needs to flow within a pipe, etc.

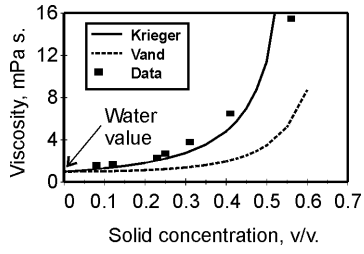


Fig. 6.15 Comparison of Newtonian viscosity equations correcting for solids presence and experimental data

The buoyancy force exerted by a fluid is the upthrust exhibited by the weight of the fluid displaced, see Archimedes' principle on page 45. However, if a large object is suspended in a stable suspension, then it will experience an upthrust due to the liquid and surrounding solids combined. The effective density of the surrounding continuous phase is average density of the fluid and solids combined

$$\rho_m = C\rho_s + (1 - C)\rho \tag{6.12}$$

The buoyancy correction for a large object within a suspension is

$$\rho_s - \rho_m$$

Hence, it is possible to quickly measure the solid concentration of a suspension using a *hydrometer* to obtain the mixture density and equation (6.12) rearranged to give

$$C = \frac{\rho_m - \rho}{\rho_s - \rho} \tag{6.13}$$

However, equation (6.12) is only valid for objects that experience the surrounding suspension as a continuous phase and not just the fluid component of it; i.e. objects larger than the particles in suspension.

The viscosity of a suspension depends upon the solid concentration and the nature of the solids. For suspensions exhibiting Newtonian flow behaviour a well-known correction for the presence of solids is Krieger's equation

$$\mu_e = \mu(1 - K'C)^{-\eta'/K'} \tag{6.14}$$

where μ_e is the effective Newtonian viscosity, K' is a crowding factor which is equal to $1/C_{max}$ (which is 1.56 for spheres) and η' is the

intrinsic viscosity, which is 2.5 for spheres. A comparison of the Krieger equation and an alternative is illustrated in Figure 6.15, together with experimental data obtained with mono-sized latex spheres in water.

When pumping suspensions, the pressure drop may be calculated using Newtonian flow equations, with equations (6.12) and (6.14) employed to correct for the presence of solids. However, this is only true for suspensions exhibiting Newtonian flow behaviour and most suspensions, at high concentration, exhibit non-Newtonian rheology.

6.7 non-Newtonian rheology

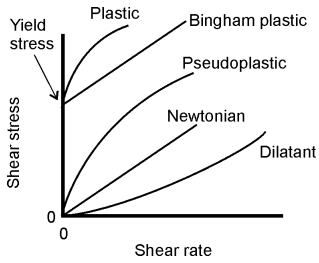


Fig. 6.16 The common time independent rheograms

Figure 6.16 illustrates the common, time-independent, rheograms. For Newtonian flow the simple relation between shear stress (R) and rate is

$$R = -\mu \frac{\mathrm{d}u}{\mathrm{d}z} \tag{6.15}$$

where μ is the coefficient of dynamic viscosity. Equation (6.15) is a single parameter model (i.e. just viscosity) to relate shear stress to rate. The next most complicated rheological model is a two parameter one

$$R = -K\left(\frac{\mathrm{d}u}{\mathrm{d}z}\right)^m \tag{6.16}$$

where K is the consistency coefficient, m is the flow index and u is the fluid, or suspension, velocity. Equation (6.16) is known as the power law model and it mathematically represents both pseudoplastic and dilitant flow on Figure 6.16; corresponding to m values less than, or greater than, unity respectively.

Under laminar flow conditions it is possible to combine equation (6.16) with a force balance at a pipe wall

$$R = \frac{a\Delta P}{2L} \tag{6.17}$$

where a is the pipe radius, and derive an analytical equation for pressure drop with suspension velocity, or flow rate, in a similar way to the derivation of the Hagen-Poiseuille equation for Newtonian fluids. The resulting equation is

$$Q = \frac{m\pi a^3}{3m+1}\left(\frac{a\Delta P}{2LK}\right)^{1/m} \tag{6.18}$$

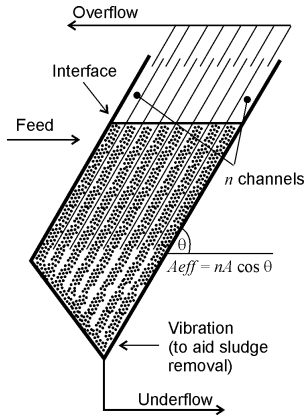


Fig. 6.17 Lamella separator or thickener – has lots of sedimentation channels in parallel

As with single phase systems, on increasing the velocity energy losses due to turbulence within the flowing suspension will become more significant. Much of the work on power law fluids was published by Dodge and Metzner in the 1950 and 60's, and they derived the Generalised Reynolds number to distinguish between laminar and turbulent flow. The threshold for the onset of sufficient turbulence to invalidate the use of equation (6.18) is about 2000 and the Generalised Reynolds (Re*) number is

$$\mathrm{Re}^* = \frac{\rho_m \bar{u}^{2-m} d^m}{\frac{K}{8}\left(\frac{6m+2}{m}\right)^m} \tag{6.19}$$

For Newtonian fluids, in the turbulent flow regime, the following equation correlates the friction factor and flow Reynolds number

$$\left(\frac{f}{2}\right)^{-1/2} = 2.5 \ln\left(\mathrm{Re}\left(\frac{f}{2}\right)^{1/2}\right) + 0.3 \tag{6.20}$$

The analogous correlation for power law fluids, and suspensions, is

$$(f)^{-1/2} = \frac{4}{m^{0.75}} \ln\left(\mathrm{Re}^* f^{(1-m/2)}\right) - \frac{0.4}{m^{1.2}} \tag{6.21}$$

where the friction factor is related to the wall shear stress by

$$\frac{f}{2} = \frac{R}{\rho \bar{u}^2} \tag{6.22}$$

6.8 Summary

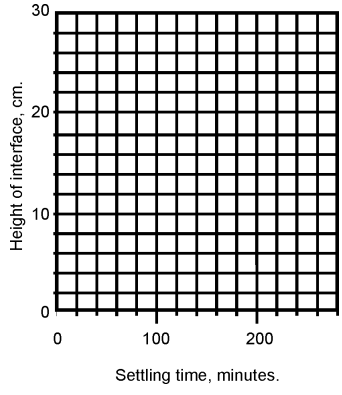
If a continuous thickener is designed correctly sufficient area will be present to allow the solids to settle into the underflow, whilst supernatant liquid reports to the overflow. If the total solids flux is in excess of the design value, the thickener will have insufficient capacity for the throughput required. Suspension, at the critical concentration, will build up inside the thickener and eventually overflow. Above the hindered settling region there may be some free settling (clarification) or simply a region of clear liquid. Some special designs exist that enhance the throughput, such as the lamella separator illustrated in Figure 6.17, but the principles of hindered settling are similar to those described in the earlier sections – the designs provide an enhanced effective plan area for the separation.

Equation (6.11) suggests that it is possible to use permeability, covered in Chapter 3, to deduce the settling rate (U_o) given that for incompressible (zone settling) the solids stress gradient will be zero. This is true, and it is possible to rearrange equation (6.11) for settling velocity and combine with equation (6.3), for flux, or equation (6.9) for plan area of a thickener. If the Kozeny-Carman model of permeability is used, however, it is usual to reduce the Kozeny constant from the usual value of 5 to 3.3, as this is believed to give a better fit to the data. However, there is a wealth of evidence to support this so-called constant being a function of solids concentration.

6.9 Problems

1.

i). A 3% v/v suspension is to be settled in a batch settling tank prior to water reuse within a process. A sample of the suspension settled



with a clear interface when placed in a 1000 ml measuring cylinder. The batch settling curve was as follows, plot the data on the grid, or graph paper.

Time (mins):	0	20	40	60	80	100	120	140	160	200	240	280
Height (cm):	28	24.2	20.4	16.5	13.6	11.2	9.4	8.1	6.8	5.4	4.4	4.0

ii). It is proposed to operate the settling tank on a three day cycle, in which 1000 m^3 of suspension is run into the tank each day, allowed to settle and some of the liquid recycled back to the process. At the start of the third day 85% of the liquid from the first two days has been recycled. The total solid volume in the tank after addition of suspension on day 3 is (m^3):.....

The total liquid volume in the tank is (m^3):.....

the tank design is now based on what happens on day 3, on day 4 the sludge in the tank will be pumped out and the cycle started again.

iii). It may be assumed that the action of adding suspension on day 3 completely mixes the tank to give a uniform suspension leading to a tank solid concentration of (% by v/v):

a: 6.7 b: 3.0 c: 9.0 d: 7.1

iv). It is possible to write a mass balance relating the height (H_f) of a uniformly mixed suspension of one concentration (C_f) to the height (H_1) and concentration (C_1) of the *same mass* of solids but mixed to an alternative concentration, as follows:

$$C_f H_f A \rho_s = C_1 H_1 A \rho_s$$

where A is vessel area and ρ_s is the solid density. Thus, you have been given the settling data for a 3% v/v suspension above which needs to be converted into settling data at the concentration determined in Part (iii); the height (H_1) required in the above equation is (cm):

a: 28.0 b: 9.3 c: 12.6 d: 14.0

v). Take a ruler and draw a line from the height determined in Part (iv) making a tangent to the settling curve plotted in Part (i). You have now obtained a settling curve for a uniform suspension of the concentration given in Part (iii). *note that settling rate is independent of vessel diameter* (but depends strongly on suspension concentration), hence the settling curve obtained in a 1000 ml measuring cylinder will be the same as that obtained in a large process vessel.

vi). As settling rate is independent of vessel diameter one design is to construct a vessel of height given in Part (iv). The total vessel volume would need to be sufficient to accommodate the volumes given in Part (ii). This would make the vessel area (m^2):

a: 4825 b: 1070 c: 48250 d: 10700

Rheology summary for Q.4

A commonly used correlation between shear stress (R) and shear rate (γ) is the power law expression

$$R = K\gamma^{m}$$

where K is the *consistency coefficient* and m is the flow index. For laminar flow the pressure drop is related to the flow rate *via* Wilkinson's equation

$$\frac{\Delta P}{L} = \frac{2K}{a}\left[\frac{(3m+1)Q}{\pi a^3 m}\right]^m$$

where Q is the volumetric flow rate and a is the pipe radius. The generalised Reynolds number (Re*) is

$$\mathrm{Re}* = \frac{8\rho_m \bar{u}^{2-m} d^m}{K(6+2/m)^m}$$

For turbulent flow the friction factor is

$$\left(\frac{f}{2}\right)^{-1/2} = 2.5\ln\left(\mathrm{Re}\left(\frac{f}{2}\right)^{1/2}\right)$$

and, combining (6.17) & (6.22),

$$\frac{f}{2} = \frac{\Delta P d}{4L\rho\bar{u}^2}$$

vii). At the end of the settlement period, i.e. on day 4 after pumping the supernatant but before pumping the sediment out, the volume of sediment is (m^3):

a: 90 b: 527 c: 437 d: 1351

viii). Hence, given the vessel area from Part (vi), the height required for the sediment is (m):

a: 0.049 b: 129 c: 0.129 d: 0.00013

ix). Draw a line from the origin of your settling graph to meet the settling curve at the height given in Part (viii). This occurs at the time (minutes):

a: 80 b: 160 c: 220 d: 280

You have now completed one design for this settling vessel: the height comes from Part (iv), the area from Part (vi), and the time required to settle from Part (ix).

x). Comment below on your design.

xi). The line you have just drawn from the origin to the settling curve represents a *solid characteristic* at a concentration greater than that given in Part (iii). It can be used to provide another vessel design. Assuming that the maximum permissible time for settling is 24 hours, the height of this characteristic after 24 hours is (cm):

a: 12.6 b: 28.0 c: 32.0 d: 56.0

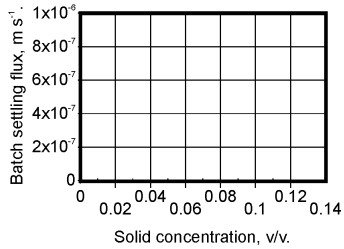
xii).Assuming that this height again represents the depth of sediment in the vessel, the new vessel area required to accommodate the total sediment on day 4 is (m^2):

a: 1650 b: 3290 c: 6590 d: 13200

xiii). Under these conditions the new vessel height will be (m):

a: 4.02 b: 2.17 c: 0.82 d: 0.28

xiv). Explain below how other characteristics can be used to provide alternative designs:



2.

i). A continuous thickener is to be designed to deal with the effluent from the last question. It will treat the 1000 m^3 per day of suspension fed at 3% v/v solids concentration and is to discharge underflow at 13.8% v/v solids. Use the settling curve from question (1) and a mass balance to complete the following table.

Concn (v/v)	0.03	0.039	0.045	0.049	0.056	0.067	0.074	0.092
Height on axis (cm)	28	21.5						
Velocity (m s^{-1})	3.2×10^{-5}							
Batch flux (m s^{-1})	9.5×10^{-7}							

note batch flux is the product of settling velocity and solid concentration

Mass balance, where C_f is any concentration in a batch settling vessel and H_f is the corresponding height:

$$C_f H_f A \rho_s = C_1 H_1 A \rho_s$$

ii). Plot the batch flux curve on the grid provided, or on graph paper.

iii). Now a flux balance on a thickener provides the following result:

$$A(TC_u) = FC_f = YC_u$$

where A is the thickener area, (TC_u) is the critical thickener flux which is the intercept of a line drawn as a tangent to the batch flux curve and going through the desired underflow concentration, F and Y are

the volume feed and underflow rates respectively, C_f and C_u are the volume fraction feed and underflow concentrations respectively.
Note that T is, in effect, the velocity of solid movement in the thickener caused by underflow withdrawal at the solid concentration C_u. The critical flux in this thickener giving an underflow discharge concentration of 13.8% v/v solids is (m s^{-1}):
a: $10x10^{-7}$ b: $8.5x10^{-7}$ c: $7.2x10^{-7}$ d: $5.8x10^{-7}$
iv). The minimum thickener area for this duty is (m^2):
a: 480 b: 29000 c: 16000 d: 960
v). If the thickener is circular in cross-section the minimum thickener diameter is (m):
a: 25 b: 190 c: 140 d: 35
vi). The underflow rate is (m^2 $hour^{-1}$):
a: 1.25 b: 2.4 c: 4.6 d: 9.1
vii). The overflow rate is (m^3 $hour^{-1}$):
a: 40.4 b: 39.3 c: 37.1 d: 32.6

3. An existing 5 m diameter thickener is to be used to thicken 2400 tonnes per 24 hours of flocculated slurry containing 10% solids by mass (0.037 v/v) in water. The solid density is 2900 kg m^3. The following batch sedimentation results were obtained in a test:

Time (mins):	0	2	4	6	8	10	12	20	30
Interface height (cm):	45.6	36.5	28.0	21.6	16.8	14.5	13.2	10.6	9.7

What will be the underflow concentration? (Ans 19% by **mass**)

4.

In laminar flow of Newtonian fluids pressure drop and flow rate are proportional – see how this suspension *shear thins*?

i). Rheological tests on *milk of magnesia* have provided the following data. Calculate the consistency coefficient and the flow index.

shear rate (s^{-1}):	7.2	16	64	320	720
shear stress (Pa):	7.0	9.1	14.3	24.2	31.6

ii). The suspension must be pumped 2.11 m down a pipe of radius 6.95 mm. Mean suspension density is 1300 kg m^{-3}. Complete the table and sketch the graph of pressure drop against flow rate. How does it differ from what is expected of a Newtonian suspension?

For Laminar flow:				For Turbulent flow:			
Pressure drop (kPa)	Flow rate (m^3 s^{-1})	Velocity (m s^{-1})	Re*	Velocity (m s^{-1})	Re*	$f^{-1/2}$ equn(6.17) & (6.22)	RHS of equn (6.20)
5			--------	--------	--------	--------	--------
15			--------	--------	--------	--------	--------
30				--------	--------	--------	--------
40				4.66			
80			--------	7.64			