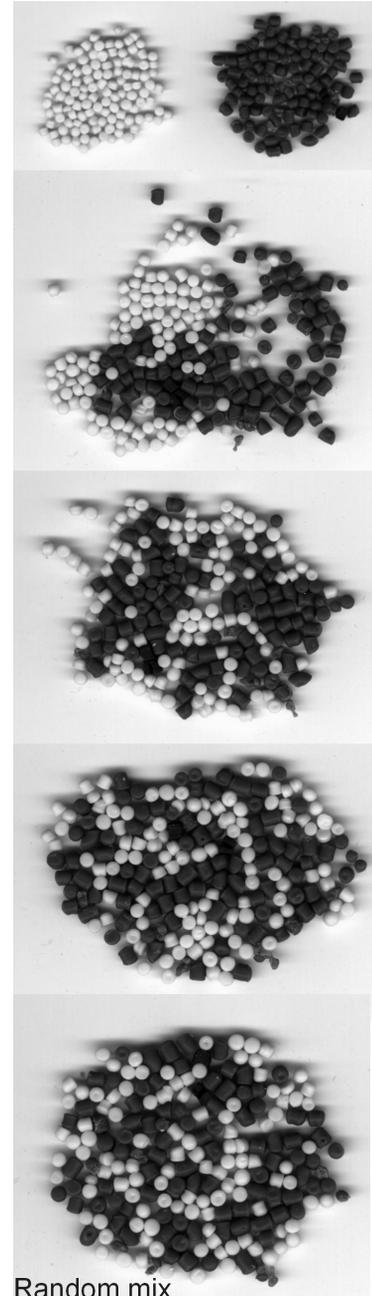


## 12 Solid/solid mixing

The mixing of solids is a critically important operation in many industries, especially in pharmaceutical production where the active ingredient in a formulation may be toxic and be present at only 0.5% by mass overall. A product with too low an active ingredient will be ineffective and a product with too high active ingredient may be lethal. To provide good solid mixing the phenomenon to be avoided, or overcome, is the particles' tendency to *segregate*. Segregation occurs when a system contains particles with different sizes, densities, etc. and motion can cause particles to preferentially accumulate into one area over another; e.g. large particles work their way to the top of breakfast cereal – fines are found at the bottom of the packet. In contrast, motion of gases and miscible liquids due to flow (convection) provides mixing on a large scale and molecular diffusion is important for completing the process at the micro-scale of mixing. Thus, removing the top off a perfume bottle in a room will result in the vapour evenly distributing itself throughout the room. A particle mixture will never be as homogeneous as that of a fluid as particles tend to segregate, whereas fluid molecules tend to mix.

In general, for particle mixing the following *physical particle properties* should be considered. Monosize particles are easy to mix, provided they are free flowing, but segregation by size, density and rotational inertia are possible with free flowing powders possessing differences by these properties. Fine particles with high surface forces (diameters  $<100\ \mu\text{m}$  and very high forces with diameters  $<10\ \mu\text{m}$ ), may need agglomerate breakage requiring high power, but can give good mixing of *cohesive* powders. *Aeration*: e.g. catalyst particles in gas fluidisation, may undergo diffusional type mixing, which is a low energy process, but with a risk of powder flooding. *Friability*: for delicate particles mixing by shear mechanisms would be inappropriate. *Explosion hazard*: an inert gas blanket is needed and low specific power input (low shear) is required. *Physiological hazard*: need to avoid airborne dust formation. *Adherence to surfaces*: easy to clean surfaces needed and if a liquid cleaning fluid is used then a new pollution problem may result.

Three mechanism types are often used to describe mixing performance: *diffusion*, but *not* molecular diffusion – an expanded bed of free flowing material occurs with particles in random movements, *convection* – when volumes, or regions, of the mix are moved en-masse to different areas, and *shear* – mixing occurs along the slip planes between regions of particles. All the mechanisms may exist in a single mixer, but one or two may predominate. The mixer type needs to be right for the material mixed, e.g. cohesive powders are more likely to require shear (and convection) hence blades and ploughs are more appropriate than tumbling.

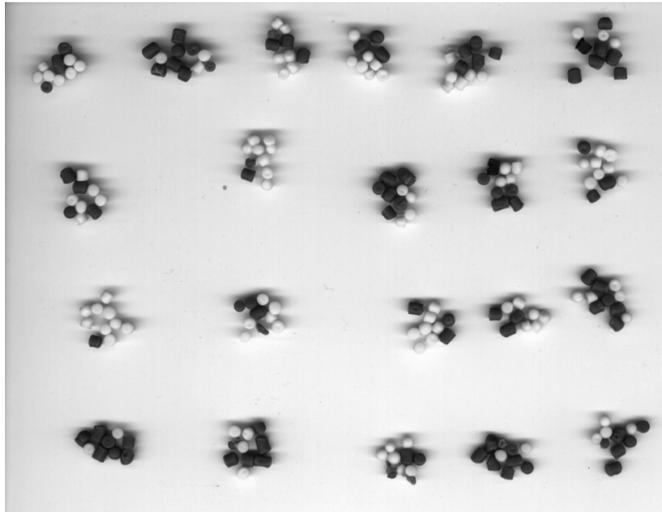


Random mix

**Fig. 12.1** Stages in mixing dark and light coloured beads to give a complete random mix

The mathematical description of the mixing process starts by considering the simple case of mixing two components differing only by colour.

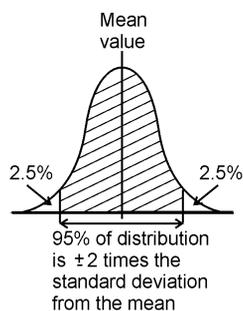
### 12.1 Binary component mixing



**Fig. 12.2** The random mix split into 21 samples and the proportion of dark chips measured in each sample: mean proportion is 0.472 with a standard deviation of 0.195

Figure 12.1 illustrates 113 light coloured chips and 101 dark coloured ones at various stages of mixing. The overall proportion of dark chips is 0.472 (i.e. 101/214) and, at any instance in time, if we were to split the mixture into many different samples and investigate the proportion of dark chips within the sample we would expect to obtain a proportion of 0.472. However, it is unrealistic to expect all the samples to have this proportion of dark chips: some samples will have more, some less, but the overall mean average of dark chips as a proportion must equal this value. Figure 12.2 illustrates

such a collection of samples, where there are 21 separate samples: the lowest proportion of dark chips is 0.1 and the highest is 0.78. The mean proportion calculated over all the samples is 0.472, the same as the overall proportion used, and the standard deviation and variance around this mean is 0.195 and 0.0382, respectively. This illustrates an important concept: the *between sample variance*. If these samples were to be sold as our product, then the variance between the products would be an important measure of the difference in quality of our product. Thus an understanding of the expected difference is important. The statistical terms and the normal distribution are briefly discussed in, and below, Figure 12.3.



( 68.2 and 99.8% of the distribution are one and three times the standard deviation from the mean )

**Fig. 12.3** The Normal probability distribution – symmetrical around the mean value

Using our knowledge of the normal distribution, and assuming that our randomly sampled coloured chips follow it, we would expect 68.2% of the samples to have a proportion of dark chips equal to 0.472 plus or minus the standard deviation, i.e.  $0.472 \pm 0.195$ . Hence, out of 21 samples 3.3 samples should be less than 0.277 and 3.3 samples more than 0.667. In Figure 12.2 we find 4 samples greater than 0.667 and 3 samples less than 0.277, in accordance with the distribution. The lowest proportion was 0.1, the highest 0.78; both are within the 95% that we expect: i.e. 2 times the standard deviation from the mean. We would need to take 40 samples to find one below 0.082 and one above 0.862.

Figure 12.4 illustrates an ordered dispersion of particles, rather than a random dispersion. This might be the degree of mixing desired, but it is very unlikely that it will be achieved by random mixing; i.e. it could only be achieved by placing the particles in order and not by a mechanical mixing process.

In powder mixing, pioneering work was by Lacy (1943, Trans. IChemE, 21, p53 & 1954, J. Appl. Chem., 4, p257), in showing that for a binary mixture, of identical particles apart from colour, the variance is

$$\sigma_R^2 = \frac{pq}{n_o} \quad (12.1)$$

where  $n_o$  is the number of particles in the sample,  $p$  and  $q$  represent number proportion of the two components (where  $p+q=1$ ),  $\bar{p}$  and  $\sigma_R^2$  are the mean and variance between the samples. Lacy also showed that the variance between samples for one component in the binary mixture in its *unmixed* state ( $\sigma_o^2$ ) will be

$$\sigma_o^2 = pq \quad (12.2)$$

This represents the worst case that one would expect. An example of the use of these equations follows. Assume that a type of children's confectionery is sold in tubes containing 100 coloured sweets, equal proportions of blue and red. In all other physical characteristics the sweets are identical. Hence, if the sweets are batch mixed before filling the tubes we should have the following situation: comparing the worst-case random variation of red sweets between tubes we would expect the variance to be

$$= 0.5 \times 0.5 = 0.25 \text{ (i.e. standard deviation } \sigma_o = 0.5)$$

and at best the variance will be

$$= 0.5 \times 0.5 / 100 = 0.0025 \text{ (std. deviation } \sigma_R = 0.05)$$

Now, to put these values into context. If we assume a normal distribution then 95% of the distribution lies  $\pm 1.96\sigma$  from the mean value. Hence, when fully mixed, out of 100 tubes of sweets we would expect to find 95 with a proportion of 0.5 reds  $\pm 1.96 \times 0.05$ , i.e. 50 reds  $\pm 9.8$  sweets. For the sake of rounding let's call this  $2\sigma$ , and 50 reds plus or minus 10 sweets. Five tubes contain sweets outside of this limit: 2.5 tubes with less than 40 reds and 2.5 tubes with more than 60 reds. Similarly, using the 99.8% confidence limit ( $3.06\sigma$ ) we have: 0.5 reds  $\pm 3.06 \times 0.05 = 0.653$  hence out of 1000 tubes 1 tube will have more than 65.3 red sweets and 1 tube will have less than 34.7. In summary, out of 1000 tubes 25 tubes have less than 40 red sweets and 1 tube has less than 35. Similarly, 25 tubes have more than 60 red sweets and 1 tube has more than 65. This assumes a normal distribution and no bias in mixing or filling the tubes. The idealised perfect mix is all tubes containing 50 red sweets but this cannot be obtained by random mixing, only by structured mixing (i.e. counting

#### Basic definitions

Mean:

$$\bar{p} = (p_1 + p_2 \dots + p_n) / n_o$$

Standard deviation:

$$\sigma_R = \sqrt{\frac{\sum (p - \bar{p})^2}{n_o - 1}}$$

or divide by  $n_o$  for values greater than 30.

Variance:

$$= \sigma_R^2$$

For a normal distribution the mean = median = mode

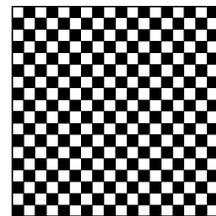


Fig. 12.4 An ordered mixture - not realistically possible by random mixing

and selecting particles during filling). Now, if our mixer doesn't provide adequate random mixing then the number of tubes outside the above limits will be even greater; hence we have calculated the *best case* random mixing based on particle properties alone. Tests may be required to establish which mixer provides the best random mixing and how quickly it is achieved.

In industry it is common to weigh materials to be mixed: this is fast, accurate and convenient. Hence, the work of Lacy was extended to provide the random variation using masses, first by Stange in 1954

$$\sigma_R^2 = \frac{PQ}{M} \left[ P\bar{w}_Q \left( 1 + \frac{\sigma_Q^2}{\bar{w}_Q^2} \right) + Q\bar{w}_P \left( 1 + \frac{\sigma_P^2}{\bar{w}_P^2} \right) \right] \tag{12.3}$$

where  $P, Q$ , are mass fractions of two components,  $M$  is the mass of sample taken,  $\bar{w}_p$  and  $\bar{w}_Q$  are the mean mass by *number* of particles in the two components, i.e.

$$\bar{w}_P = \frac{\pi}{6} \rho_s \int_{x-\min}^{x-\max} x^3 n_0(x) dx \tag{12.4}$$

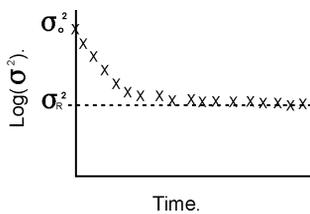
and  $\sigma_Q$  and  $\sigma_P$  are the standard deviations of the distribution of weights. This equation was later (1964) simplified by Poole, Taylor and Wall, based on mass distributions

$$\sigma_R^2 = \frac{PQ}{M} (P\bar{W}_Q + Q\bar{W}_P) \tag{12.5}$$

where  $\bar{W}_p$  and  $\bar{W}_Q$  are the mean mass by *weight* of particles in the two components. i.e.

$$\bar{W}_P = \frac{\pi}{6} \rho_s \int_{x-\min}^{x-\max} x^3 n_3(x) dx \tag{12.6}$$

assuming spherical particles and a homogeneous solid density. The resulting variance and, therefore, standard deviation is that for the variation around the mean value by mass proportion assuming a normal distribution. Equation (12.5) may be extended to provide information on mixtures containing more than two components by creating a pseudo-two component mixture; i.e. considering one of the components of interest against all the other components aggregated together. However, to do this the value of the weight weighted mean particle mass of the other components must be calculated using the mean masses of its components combined together in the correct mass proportions in which they are present.



**Fig. 12.5** How the variance changes during mixing

## 12.2 Specification and confidence

In the example of sweets we may have needed to meet a certain *specification* for our product; let's say that customers are prepared to accept some variation in the product, up to a point, and that is 15 red sweets around the mean value. This would be our specification: 50 ±15 red sweets in each tube. Our *confidence* in meeting this specification is 99.8%; we know that 1 tube will be below this limit, and 1 above, from 1000 tubes. Unfortunately, in random powder mixing we can never be 100% confident of meeting a specification. We can use the basic equations to tell us how we should minimise the number of off-spec products, and we can adopt different mixing strategies to help: e.g. add liquids to create a cohesive mixture that is not so subject to these random variations. However, all these strategies have disadvantages; e.g. cohesive mixtures take a lot of effort to mix.

We have seen already that mixture quality can be quantitatively assessed using Lacy's work on the random mixture variance ( $\sigma_R$ ) and the unmixed variance ( $\sigma_o$ ). However, we may need to know how good a mixture is between these limits. It is possible to define a mixing index  $M$ , which is equal to unity at the start of an unmixed process and approaches zero when perfectly randomly mixed

$$M = \frac{\sigma^2 - \sigma_R^2}{\sigma_o^2 - \sigma_R^2} \quad (12.7)$$

However, there are at least 30 more definitions of mixing index, so it is important to understand whichever definition is being used. Recently the term blender efficiency ( $BE$ ) has been proposed where

$$BE = \frac{\sigma}{\sigma_o} \quad (12.8)$$

For a more comprehensive analysis of mixing indexes see Fan, L.T., et al, 1979, Powder Technology, 24, p73.

The rate of mixing is very important: equations (12.1), (12.3) and (12.5) only provide the mixture quality when fully randomly mixed. Equation (12.2) provides the starting mixture quality and the quality between these two extremes may vary in accordance with Figure 12.5, if the mixing is mainly by a mechanism that is similar to diffusion; i.e. random motion of particles at a length scale similar to the particle size. However, it is perfectly possible for the variance to rise again during mixing because of segregation.

*Specification* is what you want, and *confidence* is the likelihood of achieving it.

N.B. The variance reduces in a logarithmic way towards the random value - but segregation may cause the curve to rise up again and poor mixer selection, or operation, may mean that the random value is never attained.

Another important consideration when investigating mixing is the size of sample to take, i.e. the *scale of scrutiny*. Note, this is not the same as the size of sample required for analysis, sub-division may be required and multiple analysis of fractions. The correct sample size is the same mass that the material is being mixed for, examples include: a pharmaceutical tablet's worth, a packet of cake mixture, a cement sack one cow dinner – and not the entire sack! Hence, the sampling provides an indication of how well mixed the solids are, and can be used to follow the rate of mixing *and* the analysis may also be used to predict the variation in the final product quality.

### 12.3 Equipment

See:

[www.midlandit.co.uk/particletechnology/chapter12](http://www.midlandit.co.uk/particletechnology/chapter12) for further information on mixer types and links to suppliers web sites

It is possible to obtain a good mixer quality, but a poor product quality as there are other operations between mixing and use, e.g. emptying, transportation and use, all of which may induce segregation. In a process analysis the permissible variation in the product quality must be known and its relation to quality from the mixer deduced. Batch mixing large quantities (up to 2 tonnes) reduces labour costs but as size goes up so does time to reach desired quality, filling and emptying times per batch. Continuous mixing depends on metering rates, capacity of mixer, axial and radial dispersion performance. Loading two, or more, components together should reduce batch mixing time, but requires metering or practice. A brief equipment type description follows.

*Rotating shapes:* tumbling action that induces particles to roll and fall, material is elevated beyond its angle of repose and it falls to the free surface. Good mixing of free flowing materials (tea, seeds, etc.). Easy cleaning and emptying and power consumption and wear are low to moderate.

*Ribbon blade:* an agitator mixes material in a trough. Reasonably gentle mixing but with shear and impaction. Not suitable for very cohesive materials, unless a dough is required. Used for addition of small amounts to larger components, but can be difficult to clean.

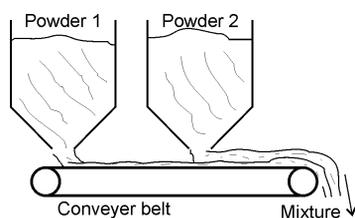
*Orbiting screw:* charge is lifted up through mixer and spread out. Can be modified for paste use. Moderate power consumption and reasonable to clean.

*Pan mixer:* blades or ploughs move through mixture with various angles of attack. Mainly found in food and pharmaceutical use, easy to clean but has a high power consumption.

*Z-blade:* two contra-rotating blades, seldom used for dry materials ideal for a dough. Very high power requirement and difficult to clean.

*Muller or edge runner mill:* crushes aggregates and mixes well, but not used for free flowing or too cohesive mixtures – just the right amount of friction is needed to resist the rollers.

*High speed impeller:* material hits impeller and is thrown out towards the wall. Very high power consumption, but rapid mixing.



**Fig. 12.6** Blending by metering correct proportions of components from storage hoppers

Others include fluidised beds and air lifts, blending from hopper (bunker) discharges and stockpile blending. Continuous mixing of free flowing powders includes metering the solids out at a continuous and well defined rate, such as on a conveyor belt, see Figure 12.6. This avoids the problem of mixing, but requires precise powder control. This is only sometimes achievable in practice. Also, this arrangements assumes that it is acceptable to form the product from two separate streams; e.g. mixing will take place in the bag or by the consumer. Metering, together with continuous mixing, has become accepted for cohesive powders, where segregation is less likely and is sometimes used with free flowing ones. Table 12.1 compares batch and continuous mixing strategies.

### 12.4 Cohesive powder mixing

When mixing bread a lot of effort is required to stir the cohesive mixture but it doesn't segregate. However, only small pockets of mixture receive shear, and mixing, and the energy input is high. In commercial systems this means a powerful motor is necessary. Cohesive mixtures tend to be formed by fine particles and systems with binders present. Smaller particles lead to a larger number of particles for a given sample mass. As the number of particles goes up the variance between samples will go down, see equation (12.1). This is intuitive, as the limit would be to go to molecular mixing (very small particles) where the variance is zero between samples. However, with powders, Lacy pointed out that the situation could arise where smaller particles actually increase the variance between samples; if a mixture contains aggregates, say of paint pigments blue and red in colour, then overall the mixture may be good but, if the sample size is equal to the aggregate size, it is possible that each aggregate could be selected. This would give a very large value for the calculated variance. So, although the primary particle size is small the particles aggregate and the true particle number within the sample is just one! Clearly, some careful interpretation of the data is required and consideration of the appropriate scale of scrutiny is most important.

### 12.5 Summary

Even when fully mixed we know that there will be a difference between samples, or products, made from a powder. This variation is given by Lacy's or Poole, Taylor and Walls' equations. If this variation is too great for our specification we must either supply components separately or decrease the size of the particles to give a more intimate mix. This may lead to powder handling problems and a strategy of size reduction, followed by mixing, then granulation before further processing may be required. In general, mixing is easier when particle size and other physical properties are the same. When a small amount of one component is mixed with a large amount of another, we may need to grind finely and coat larger particles.

Table 12.1 Comparison of continuous and batch mixing of free flowing powders: italicised description indicates preferred option

Property	Batch	Continuous
Capital cost: mixer, ancillary	higher <i>lower</i>	<i>lower</i> higher
Operating cost	automation reduce labour costs but increase capital costs	<i>fewer operators in general</i>
Flexibility	<i>More flexible</i>	one operation only
Proportion ingredients	<i>More confidence</i>	can be ok but needs analysis
Handling and storage		<i>not necessary - useful for segregating mixes</i>
Capacity		<i>much greater</i>
Quality control	<i>batch identifiable</i>	not so
Design knowledge	high	ok
Mixing efficiency	similar if operated ok	

A common misconception is that to overcome poor mixing an increase in mixing time may be applied. If segregation occurs an *increase in time will increase segregation*, therefore, mixing for longer produces a poorer dispersion. In many cases process considerations are important as well as mixing efficiency. For example, in pharmaceutical production it is important to be able to identify batch numbers; hence, batch mixing is preferred. To overcome segregation, deliberately forming a cohesive mixture by using a binder might be performed, but this will require higher energy costs than dry powder mixing and may result in more difficult to clean mixing machinery.

## 12.6 Problems

Consult equations (12.1) and (12.5), but you will need to convert between number and mass proportions. See discussion around Figure 12.3 for a discussion on the number out of *spec* tablets.

1. According to a British Pharmacopia standard, the allowable variation in composition of an active ingredient is  $\pm 10\%$ . A tablet contains 10% active ingredient and 90% inert diluent. Calculate the minimum number of particles to each tablet so that it may be within the B.P. specification (assume the standard is based on number):

- assume that the particles are all the same size, and
  - assume that the particles are size distributed so that the mean weight of a particle of the active ingredient is 300  $\mu\text{g}$ , and the other ingredient is 100  $\mu\text{g}$ .
- c) Out of a batch of 1 million tablets how many will be outside the B.P. specification. How could you improve the situation?

2. Three powders (A, B and C) are to be mixed in the proportion of 60:30:10 (A:B:C) by weight. Each powder consists of monosized spherical particles of density 2600  $\text{kg m}^{-3}$  and particles of diameter 100  $\mu\text{m}$ , 80  $\mu\text{m}$  and 50  $\mu\text{m}$  for A, B and C respectively. In the evaluation of the mixing process with respect to powder C, the between sample standard deviations of 100 g samples were obtained as a function of time at 20, 40 and 60 minutes, the corresponding standard deviations were:  $5.2 \times 10^{-4}$ ,  $1.4 \times 10^{-4}$  and  $5.6 \times 10^{-5}$ . Estimate the minimum mixing time for complete randomness. What confidence would you have in this result? Is further experimentation necessary?

3. A mix contains two components, A and B, the fractional mass undersize particle size distributions are (where  $x$  is in mm):

$$\text{A: } N_3(x) = \left(\frac{x}{0.5}\right)^{0.5} \text{ for } x \leq 0.5 \text{ mm and}$$

$$\text{B: } N_3(x) = \left(\frac{x}{0.25}\right)^2 \text{ for } x \leq 0.25 \text{ mm}$$

A and B are in mass proportions 2:3 and have densities 3500 and 2500  $\text{kg m}^{-3}$  respectively. The specification requires that in samples of 4 g the proportion of A should be within  $\pm 5\%$  of the nominal value. Comment on this.